

University of Groningen

Hankel norm approximation for infinite-dimensional systems

Sasane, Amol Jagannath

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version

Publisher's PDF, also known as Version of record

Publication date:

2001

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Sasane, A. J. (2001). *Hankel norm approximation for infinite-dimensional systems*. s.n.

Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

Summary

Model reduction is an important engineering problem, in which one aims to replace an elaborate model by a simpler model without undue loss of accuracy. The accuracy can be mathematically measured in several possible norms and the Hankel norm is one such. The Hankel norm gives a meaningful notion of distance between two linear systems: it is the induced norm of the operator that maps past inputs to future outputs. It turns out that the engineering problem of model reduction in the Hankel norm is closely related to the mathematical problem of finding solutions to the sub-optimal Nehari-Takagi problem, which we prefer to call “the sub-optimal Hankel norm approximation problem” in this thesis. Although the solution to the sub-optimal Hankel norm approximation problem has been known since the 1970’s, in this thesis, we give an alternative derivation and, in particular, we give new explicit formulae for some special classes.

The approach taken in this thesis is as follows. We start with a complex matrix-valued function G defined on the imaginary axis satisfying certain assumptions. In particular, we demand the existence of a solution to a certain J -spectral factorization problem. We then give a solution to the sub-optimal Hankel norm approximation problem and furthermore, we give a parameterization of all solutions in terms of the J -spectral factor, the parameterizing set being the unit ball in a certain Hardy space. In this manner we give a purely “frequency domain” solution to the sub-optimal Hankel norm approximation problem in Chapter 4 of this thesis.

In the subsequent chapters (6 and 7) we consider the case where G is in fact the transfer function of certain classes of infinite-dimensional well-posed linear systems given by a triple of operators (A, B, C) . We solve the sub-optimal Hankel norm approximation problem for such G ’s by constructing a J -spectral factor in terms of the system parameters (A, B, C) and verify that this constructed J -spectral factor satisfies the assumptions demanded in Chapter 4. In particular, in order to verify an assumption about the number of unstable poles of the inverse of a certain sub-block of the J -spectral factor, we prove an inertia theorem for operator Lyapunov equations in Chapter 5. In

this manner, we give a “state-space” solution to the sub-optimal Hankel norm approximation problem for two important classes of well-posed linear systems: the smooth Pritchard-Salamon class of exponentially stable infinite-dimensional systems and the class of exponentially stable analytic systems. Furthermore, in the final chapter of this thesis, we also solve the sub-optimal Hankel norm approximation problem for a certain class of infinite-dimensional systems with a non-exponentially stable semigroup.